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ABSTRACT

This study examined the effects of number of illustrative examples and number of practice problems given on seventh graders' achievement in a unit on probability. Four treatment groups were used, one for each combination of two levels of number of examples and two levels of number of practice problems. Prior to treatment students were pretested on the Missing Words test, the Necessary Arithmetical Operations test, computation with rational numbers, and probability. Tests given during and after the programmed treatments concerned computation, relevant and irrelevant attributes of examples, and probability topics. Subscale scores for each test were analyzed using analysis of covariance. Marginally significant results indicated that practice had some effect on relevant and irrelevant concept scales on the first test. The author states that further research on these variables is needed. Computer-generated regression and analysis of covariance tables are appended, as are copies of the tests. (SD)

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SMESG Working Paper No. 7

Effects of Varying the Number of Examples and Practice Problems

By: Barbara Pence and E. G. Begle

Introduction

This paper reports a study in which both the variables listed in the title were manipulated. Both these variables have been investigated in concept formation studies in which a subject, after examining a set of stimuli, trys to guess the concept in the experimenter's mind. However, little research exists relating these two instructional variables to classroom learning.

The experiment was carried out in the spring of 1974 with the cooperation of the Cupertino Union School District.

As usual, programmed instruction was used in order to avoid the confounding effect due to teachers.

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Design

In this study, four programmed versions of a unit on probability were used. One version was constructed for each cell of the following 2 × 2 factorial:

		Practice Problems: Few	Practice Problems: Many
Examples of Concepts	Few	FF	FM
	Many	MF	MM

The four treatment groups were:

Group 1 - Many examples and Many practice
Group 2 - Many examples and Few practice
Group 3 - Few examples and Many practice
Group 4 - Few examples and Few practice

The two instructional variable had distinct meanings. "Example" referred to the illustrative examples provided during the explanation of a skill, concept, or principle. "Practice" applied to the practice problems which followed the explanatory discussion.

Theoretically "few" was defined as less than or equal to half the number of illustrations encountered in the many version. However, the actual program variations were a function of the concept and its role in the development of the student's ability to compute probabilities. A summary of the number of illustrative examples in the few and many versions as well as the number of practice problems in both versions is given in Appendix 2. The numbers are misleading in the case of illustrative examples because the examples required a variety of student responses from reading, to filling in blanks, to producing short answers. In all cases, the illustrative examples were a part of the development. Consequently, one additional example might involve as much as a page of additional illustrative discourse.

Pretests. In order to take individual differences into account, a short battery of pretests was administered. The four tests were:

Missing Words - A mathematics reading test discussed in SMESG Working Papers No. 1 and 2.

Necessary Arithmetic Operations - A Test taken from French's Kit of reference tests for cognitive factors.

Computation with Rationals - NLSMA Scale X307.

Probability - A test of entry knowledge of text material.

Posttests. The programmed text contained three sections. Section 1 stressed the development of concepts. Section 2 emphasized the computational algorithm. Section 3 provided supplementary material for the faster students.

Tests were given after Sections 1 and 2. Each test was divided into scales. Test 1 contained the three scales of relevant dimension of a concept, irrelevant dimensions of a concept, and computation. (For example, for the concept "triangle", the number of sides of a geometric figure would be a relevant dimension, while the size of a figure would be irrelevant.) In test 2, the two scales were relevant dimensions and computation. Appendix 3 identifies the topics, relevant dimensions, irrelevant dimensions, and test items by topic. Item assignment by test scale is also reported. Appendix 4 contains the actual tests.

Subjects. Eight seventh grade classes from three schools participated in the study. Assignment to treatment was made randomly within each class. The number of students in each group taking Test 1 was:

Group 1:	37	Group 3:	35
Group 2:	37	Group 4:	33

The number of students taking Test 2 were:

Group 1:	14	Group 3:	15
Group 2:	21	Group 4:	23

Procedures. The classroom teachers administered the study according to step by step instructions.

Results. Table 1 gives the item analysis for each of the three scales of Test 1. In analyzing the data for treatment differences the probability pretest was eliminated because the mean was less than the chance level. Also dropped from consideration were students missing scores on one or more of the pretests. The means, standard deviations, and correlation matrix for the pretests and the scales of achievement for Test 1 for the four separate treatments are shown in Table 2.

In order to take initial differences between groups into account, the analysis of covariance was used. Tables 3-5 show the regression equations, the ANCOVA table, and the table of adjusted means for each scale of achievement test 1. None of the scales reflect significant differences, but the relevant and irrelevant scales have suspiciously low p-values.

Table 6 provides the ANCOVA and adjusted means for each scale in the comparison of few example with many examples. Table 7 gives the same information for the contrast between few and many practice exercises. Practice made a difference in the relevant concept scale, $p < .06$. Both examples and practice affected the irrelevant concept scale, $p < .09$ and $p < .11$ respectively.

In a further attempt to isolate the source of variance, the data was sorted on the Necessary Arithmetic Operations score. Since the mean score was 6.20, seven was chosen as the boundary value. All cases with scores below 7 were placed in the "low" group while cases with scores of 7 or higher were placed in the "high" group.

Table 8 summarizes the ANCOVA and adjusted means for the low group across treatments. Table 9 provides the results for the comparison of variations in the number of examples and Table 10 shows the effect of variations in the number of practice problems.

Tables 11-13 provide the same information for the high group.

For the low group, many practice problems produced significantly higher scores on the relevant concept scale, $p < .03$. Although none of

the experimental contrasts were significant for the high group, the irrelevant scale scores were higher for the many version for both examples and practice, $p < .15$ and $p < .06$ respectively.

The second achievement test contained only two scales. Item analyses for these scales are reported in Table 14. Table 15 gives the descriptive statistical data for each group separately.

The ANCOVA across all treatments is given in Table 16. Analyses for contrasts between few and many examples are reported in Table 17 and the results of comparing few and many practice exercises are shown in Table 18. The sample size for Test 2 was not large enough to justify any additional subdivisions of the sample.

Notice that both scales reflected low p-values across all four treatments. However, when the scales were analyzed by variable, there was only one contrast with a low p-value, $p < .07$. On the computation scale, the adjusted means for many examples exceeded the adjusted means for few examples.

The following table summarizes the results of the various analyses.

Summary of "p" values
for Experimental Treatments
Test 1

Treatment/Population	All	Low	High
Examples			
Relevant	1.000	.665	.779
Irrelevant	.083	.444	.144
Computation	.855	.582	.734
Practice			
Relevant	.056	.027	.935
Irrelevant	.110	.626	.055
Computation	.235	.243	.830
All Treatments			
Relevant	.163	.036	.987
Irrelevant	.129	.668	.118
Computation	.520	.235	.854

Test 2

Examples	All	Low ⟵ N ⟶ High
Relevant	.254	
Computation	.079	wasn't
Practice		large
Relevant	.337	enough
Computation	.915	for
All Treatments		stratification
Relevant	.008	
Computation	.120	

Conclusions

The distribution of small p values in the above table makes it clear that both these variables should be investigated further, using other mathematical topics and also other grade levels.

APPENDIX 1

TABLE 1

SCALE STATISTICS: RELEVANT CONCEPT

NUMBER OF CASES	=	150
NUMBER OF ITEMS	=	19
MEAN TOTAL SCORE	=	10.380
STANDARD DEVIATION	=	3.076
CRONBACH'S ALPHA	=	0.644
ERROR OF MEASUREMENT	=	1.836

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
1	0.380	0.385	0.153	1.333
2	0.533	0.533	0.080	0.0
3	0.720	0.740	0.343	2.667
7	0.160	0.163	-0.283	2.000
16	0.607	0.619	0.464	2.000
14	0.407	0.418	0.195	2.667
15	0.793	0.815	0.516	2.667
23	0.787	0.814	0.910	3.333
24	0.787	0.814	0.350	3.333
19	0.107	0.110	-0.063	2.667
20	0.533	0.580	0.265	8.000
26	0.780	0.801	0.478	2.667
27	0.440	0.455	0.022	3.333
21	0.753	0.769	0.514	2.000
25	0.713	0.743	0.723	4.000
31	0.587	0.607	0.239	3.333
32	0.127	0.132	0.150	4.000
33	0.360	0.372	0.266	3.333
34	0.807	0.840	0.829	4.000

SCALE STATISTICS: IRRELEVANT CONCEPT

NUMBER OF CASES	=	150
NUMBER OF ITEMS	=	10
MEAN TOTAL SCORE	=	4.260
STANDARD DEVIATION	=	1.397
CRONBACH'S ALPHA	=	0.019
ERROR OF MEASUREMENT	=	1.384

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
4	0.533	0.537	-0.077	0.667
5	0.173	0.176	0.034	1.333
6	0.107	0.108	-0.342	1.333
8	0.600	0.600	0.117	0.0
11	0.260	0.260	0.144	0.0
12	0.573	0.577	-0.039	0.667
24	0.787	0.819	-0.115	4.000
28	0.587	0.611	0.190	4.000
19	0.107	0.110	-0.218	2.667
20	0.533	0.576	0.132	7.333

SCALE STATISTICS: COMPUTATION

NUMBER OF CASES	=	150
NUMBER OF ITEMS	=	15
MEAN TOTAL SCORE	=	9.573
STANDARD DEVIATION	=	3.140
CRONBACH'S ALPHA	=	0.733
ERROR OF MEASUREMENT	=	1.623

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
9	0.913	0.926	0.650	1.333
10	0.833	0.850	0.702	2.000
12	0.580	0.584	0.224	0.667
13	0.833	0.845	0.065	1.333
22	0.327	0.340	0.251	4.000
28	0.587	0.611	0.551	4.000
29	0.647	0.678	0.472	4.667
30	0.507	0.535	0.326	5.333
17	0.740	0.760	0.594	2.667
18	0.753	0.774	0.533	2.667
35	0.567	0.612	0.363	7.333
36	0.500	0.528	0.230	5.333
37	0.527	0.572	0.597	8.000
38	0.547	0.621	0.615	12.000
39	0.713	0.748	0.679	4.667

TABLE 2

RAW SCORE
MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
MISSING WORD 1	2	3.38	3.38	3.23	3.12	3.28
MISSING WORD 2	3	3.25	3.54	5.03	3.33	3.81
MISSING WORD 3	4	2.84	2.89	2.60	2.70	2.76
MISSING WORD 4	5	3.46	3.70	3.54	3.91	3.65
MISSING WORD 5	6	16.78	17.68	19.14	17.48	17.76
ARITH OPERATION	7	5.84	6.14	6.46	6.39	6.20
COMPUTATION - RATI	8	11.05	10.54	11.46	10.97	11.00
ACHIEVEMENT TEST 1	9	21.89	21.24	22.60	20.09	21.48
ACHIEV REL CONCEPT	10	10.41	10.03	10.97	9.55	10.25
ACHIEV IRREL CONCE	11	4.62	4.24	4.14	3.88	4.23
ACHIEV COMPUTATION	12	9.59	9.49	10.06	9.09	9.56

STANDARD DEVIATIONS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
MISSING WORD 1	2	0.76	0.89	0.84	0.74	0.81
MISSING WORD 2	3	0.98	0.80	10.29	1.05	5.17
MISSING WORD 3	4	1.07	1.31	1.01	1.26	1.16
MISSING WORD 4	5	1.66	1.75	1.92	1.74	1.76
MISSING WORD 5	6	8.01	7.28	7.50	8.55	7.80
ARITH OPERATION	7	2.32	2.49	1.88	2.49	2.29
COMPUTATION - RATI	8	5.10	5.30	4.99	5.07	5.08
ACHIEVEMENT TEST 1	9	5.75	6.43	5.25	7.11	6.16
ACHIEV REL CONCEPT	10	3.40	3.38	2.83	3.50	3.29
ACHIEV IRREL CONCE	11	1.59	1.64	1.12	1.56	1.50
ACHIEV COMPUTATION	12	2.75	3.35	2.92	3.92	3.24

CORRELATION MATRIX FOR TOTAL

	2	3	4	5	6	7	8	9	10	11	12
2	1.00	0.13	0.35	0.34	0.30	0.19	0.25	0.34	0.28	0.21	0.31
3		1.00	0.16	0.09	0.12	0.03	0.07	0.04	0.05	0.01	0.02
4			1.00	0.42	0.38	0.33	0.44	0.38	0.26	0.35	0.39
5				1.00	0.49	0.44	0.44	0.46	0.41	0.35	0.38
6					1.00	0.49	0.45	0.33	0.28	0.15	0.32
7						1.00	0.45	0.39	0.29	0.23	0.39
8							1.00	0.51	0.42	0.43	0.45
9								1.00	0.88	0.69	0.89
10									1.00	0.52	0.61
11										1.00	0.56
12											1.00

SAMPLE SIZE = 142

DEPENDENT VARIABLE -- RELEVANT CONCEPT

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STANDARDIZED REGRESSION WEIGHTS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	POOLED	TOTAL
MISSING WORD 1	2	0.132	0.108	0.032	0.515	0.124	0.134
MISSING WORD 2	3	1.127	-1.459	-0.012	0.300	-0.017	0.000
MISSING WORD 3	4	-0.448	0.274	-0.111	-0.076	-0.002	-0.015
MISSING WORD 4	5	0.709	0.016	0.278	0.010	0.268	0.240
MISSING WORD 5	6	0.009	-0.070	0.044	0.052	-0.036	-0.018
ARITH OPERATION	7	-0.087	0.101	-0.253	0.295	0.047	0.046
COMPUTATION - RATI	8	0.384	0.351	0.113	0.090	0.258	0.271

RAW SCORE REGRESSION WEIGHTS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	POOLED	TOTAL
MISSING WORD 1	2	0.536	0.438	0.131	2.092	0.504	0.544
MISSING WORD 2	3	0.719	-0.931	-0.008	0.191	-0.011	0.000
MISSING WORD 3	4	-1.273	0.778	-0.315	-0.216	-0.006	-0.042
MISSING WORD 4	5	1.330	0.031	0.522	0.018	0.502	0.451
MISSING WORD 5	6	0.004	-0.029	0.019	0.022	-0.015	-0.008
ARITH OPERATION	7	-0.124	0.145	-0.363	0.423	0.068	0.066
COMPUTATION - RATI	8	0.249	0.228	0.074	0.059	0.167	0.176
RSQ		0.609	0.367	0.203	0.456	0.263	0.254
R		0.781	0.606	0.450	0.675	0.512	0.504
F		6.460	2.406	0.982	2.988	6.815	6.521
P		0.000	0.045	0.464	0.020	0.000	0.000

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ANALYSIS OF COVARIANCE

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SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	388.846	7.	55.549	7.155	0.000
TREATMENT MEANS	40.470	3.	13.490	1.738	0.163
HETEROGENEITY OF REGRESSION	247.048	21.	11.764	1.515	0.087
ERROR	854.011	110.	7.764		
TOTAL	1530.375	141.			

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ADJUSTED MEANS VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
ACHIEV IRREL CONCE	11	4.60	4.24	4.21	3.83	4.23

TABLE 4

DEPENDENT VARIABLE -- IRRELEVANT CONCEPT

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STANDARDIZED REGRESSION WEIGHTS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	POOLED	TOTAL
MISSING WORD 1	2	0.073	-0.033	0.030	0.254	0.026	0.052
MISSING WORD 2	3	-0.668	-0.622	-0.014	0.506	-0.037	-0.042
MISSING WORD 3	4	-0.122	0.282	0.141	0.217	0.161	0.174
MISSING WORD 4	5	0.657	0.127	0.089	0.179	0.226	0.201
MISSING WORD 5	6	-0.068	-0.078	-0.457	-0.272	-0.178	-0.180
ARITH OPERATION	7	0.177	-0.180	0.086	0.039	0.029	0.013
COMPUTATION - RATI	8	0.360	0.454	0.401	0.248	0.326	0.334

RAW SCORE REGRESSION WEIGHTS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	POOLED	TOTAL
MISSING WORD 1	2	0.135	-0.062	0.056	0.471	0.047	0.097
MISSING WORD 2	3	-0.194	-0.181	-0.004	0.147	-0.011	-0.012
MISSING WORD 3	4	-0.158	0.364	0.182	0.281	0.208	0.225
MISSING WORD 4	5	0.561	0.109	0.076	0.153	0.193	0.171
MISSING WORD 5	6	-0.013	-0.015	-0.088	-0.052	-0.034	-0.035
ARITH OPERATION	7	0.116	-0.118	0.056	0.025	0.019	0.008
COMPUTATION - RATI	8	0.106	0.134	0.115	0.073	0.096	0.099
RSQ		0.567	0.261	0.399	0.340	0.272	0.264
R		0.753	0.511	0.632	0.583	0.521	0.513
F		5.427	1.462	2.562	1.839	7.143	6.855
P		0.000	0.220	0.037	0.124	0.000	0.000

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ANALYSIS OF COVARIANCE

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SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	83.673	7.	11.953	7.017	0.000
TREATMENT MEANS	9.853	3.	3.284	1.928	0.129
HETEROGENEITY OF REGRESSION	36.410	21.	1.734	1.018	0.449
ERROR	187.395	110.	1.704		
TOTAL	317.331	141.			

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ADJUSTED MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
ACHIEV REL CONCEPT	10	10.45	10.03	10.99	9.48	10.25

TABLE 5

DEPENDENT VARIABLE -- COMPUTATION

STANDARDIZED REGRESSION WEIGHTS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	POOLED	TOTAL
MISSING WORD 1	2	0.082	0.248	0.004	0.441	0.135	0.140
MISSING WORD 2	3	-0.010	-0.868	-0.066	-0.352	-0.066	-0.053
MISSING WORD 3	4	-0.008	0.151	0.264	0.041	0.158	0.147
MISSING WORD 4	5	0.237	0.273	-0.048	0.030	0.111	0.094
MISSING WORD 5	6	0.011	-0.352	0.006	0.215	-0.031	-0.018
ARITH OPERATION	7	0.406	-0.049	-0.056	0.196	0.175	0.176
COMPUTATION - RATI	8	0.097	0.450	0.325	0.178	0.237	0.246

RAW SCORE REGRESSION WEIGHTS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	POOLED	TOTAL
MISSING WORD 1	2	0.328	0.592	0.015	1.764	0.541	0.559
MISSING WORD 2	3	-0.007	-0.545	-0.041	-0.221	-0.042	-0.033
MISSING WORD 3	4	-0.022	0.421	0.737	0.115	0.441	0.411
MISSING WORD 4	5	0.438	0.504	-0.088	0.056	0.205	0.173
MISSING WORD 5	6	0.005	-0.147	0.002	0.089	-0.013	-0.007
ARITH OPERATION	7	0.574	-0.069	-0.080	0.276	0.247	0.248
COMPUTATION - RATI	8	0.062	0.288	0.207	0.114	0.152	0.157
RSQ		0.483	0.445	0.260	0.421	0.314	0.309
R		0.695	0.667	0.510	0.649	0.560	0.556
F		3.875	3.327	1.358	2.599	8.750	8.564
P		0.004	0.010	0.263	0.037	0.000	0.000

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	458.364	7.	65.481	8.294	0.000
TREATMENT MEANS	17.936	3.	5.979	0.757	0.520
HETEROGENEITY OF REGRESSION	138.237	21.	6.583	0.834	0.673
ERROR	868.392	110.	7.894		
TOTAL	1482.930	141.			

ADJUSTED
MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
ACHIEV COMPUTATION	12	9.60	9.44	10.11	9.08	9.56

TABLE 6

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	388.846	7.	55.549	6.402	0.000
TREATMENT MEANS	-0.001	1.	-0.001	-0.000	1.000
HETEROGENEITY OF REGRESSION	48.209	7.	6.887	0.794	0.594
ERROR	1093.321	126.	8.677		
TOTAL	1530.375	141.			

DEPENDENT VARIABLE -- ACHIEV IRREL CONCE

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	83.673	7.	11.953	7.016	0.000
TREATMENT MEANS	5.184	1.	5.184	3.043	0.083
HETEROGENEITY OF REGRESSION	13.818	7.	1.974	1.159	0.331
ERROR	214.656	126.	1.704		
TOTAL	317.331	141.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	458.364	7.	65.481	8.405	0.000
TREATMENT MEANS	0.261	1.	0.261	0.034	0.855
HETEROGENEITY OF REGRESSION	42.682	7.	6.097	0.783	0.603
ERROR	981.623	126.	7.791		
TOTAL	1482.930	141.			

ADJUSTED MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	TOTAL
ACHIEV REL CONCEPT	10	10.24	10.25	10.25
ACHIEV IRREL CONCE	11	4.42	4.03	4.23
ACHIEV COMPUTATION	12	9.52	9.61	9.56

TABLE 7

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	388.846	7.	55.549	6.878	0.000
TREATMENT MEANS	30.089	1.	30.089	3.726	0.056
HETEROGENEITY OF REGRESSION	93.857	7.	13.408	1.660	0.125
ERROR	1017.583	126.	8.076		
TOTAL	1530.375	141.			

DEPENDENT VARIABLE -- ACHIEV IRREL CONCE

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	83.673	7.	11.953	6.778	0.000
TREATMENT MEANS	4.562	1.	4.562	2.587	0.110
HETEROGENEITY OF REGRESSION	6.905	7.	0.986	0.559	0.788
ERROR	222.191	126.	1.763		
TOTAL	317.331	141.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	458.364	7.	65.481	8.319	0.000
TREATMENT MEANS	11.141	1.	11.141	1.415	0.235
HETEROGENEITY OF REGRESSION	21.700	7.	3.100	0.394	0.904
ERROR	991.724	126.	7.871		
TOTAL	1482.930	141.			

ADJUSTED MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	TOTAL
ACHIEV REL CONCEPT	10	10.71	9.77	10.25
ACHIEV IRREL CONCE	11	4.41	4.05	4.23
ACHIEV COMPUTATION	12	9.84	9.28	9.56

TABLE 8

DEPENDENT VARIABLE — ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	247.698	7.	35.385	4.499	0.001
TREATMENT MEANS	72.645	3.	24.215	3.079	0.036
HETEROGENEITY OF REGRESSION	225.268	21.	10.727	1.364	0.183
ERROR	393.280	50.	7.866		
TOTAL	938.891	81.			

DEPENDENT VARIABLE -- ACHIEV IRREL CONCE

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	41.304	7.	5.901	3.127	0.009
TREATMENT MEANS	2.963	3.	0.988	0.523	0.668
HETEROGENEITY OF REGRESSION	25.367	21.	1.209	0.641	0.867
ERROR	94.358	50.	1.887		
TOTAL	164.012	81.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	226.271	7.	32.339	3.525	0.004
TREATMENT MEANS	40.346	3.	13.449	1.466	0.235
HETEROGENEITY OF REGRESSION	90.608	21.	4.315	0.470	0.969
ERROR	458.074	50.	9.173		
TOTAL	816.000	81.			

ADJUSTED MEANS VARIABLES BY GROUPS

VARIABLE NAME	NO.	MANYMANY	MANYFEW	FEWMANY	FEWFEW	TOTAL
ACHIEVEMENT TEST2	9	20.47	20.11	23.21	18.08	20.54
ACHIEV REL CONCEPT	10	10.07	9.54	11.52	8.57	9.95
ACHIEV IRREL CONCE	11	4.16	4.25	4.21	3.72	4.11
ACHIEV COMPUTATION	12	8.72	8.94	10.24	9.04	9.00

TABLE 9

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	247.696	7.	35.385	3.938	0.001
TREATMENT MEANS	1.705	1.	1.705	0.190	0.665
HETEROGENEITY OF REGRESSION	96.446	7.	13.778	1.533	0.171
ERROR	593.041	66.	8.985		
TOTAL	938.891	81.			

DEPENDENT VARIABLE -- ACHIEV IRREL CONCE

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	41.304	7.	5.901	3.488	0.003
TREATMENT MEANS	1.005	1.	1.005	0.594	0.444
HETEROGENEITY OF REGRESSION	10.063	7.	1.438	0.850	0.551
ERROR	111.840	66.	1.692		
TOTAL	164.012	81.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	226.371	7.	32.339	3.863	0.001
TREATMENT MEANS	2.558	1.	2.558	0.306	0.582
HETEROGENEITY OF REGRESSION	34.594	7.	4.942	0.590	0.761
ERROR	552.477	66.	8.371		
TOTAL	816.000	81.			

ADJUSTED MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	MANYEXAM	FEWEXAM	TOTAL
ACHIEVEMENT TEST2	9	20.34	20.81	20.54
ACHIEV REL CONCEPT	10	9.84	10.13	9.96
ACHIEV IRREL CONCE	11	4.21	3.98	4.11
ACHIEV COMPUTATION	12	8.89	9.21	9.00

TABLE 10

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	247.698	7.	35.385	4.050	0.001
TREATMENT MEANS	44.702	1.	44.702	5.116	0.027
HETEROGENEITY OF REGRESSION	69.774	7.	9.968	1.141	0.349
ERROR	576.718	66.	8.738		
TOTAL	938.891	81.			

DEPENDENT VARIABLE -- ACHIEV IRREL CONCE

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	41.304	7.	5.901	3.338	0.004
TREATMENT MEANS	0.424	1.	0.424	0.240	0.626
HETEROGENEITY OF REGRESSION	5.612	7.	0.802	0.454	0.864
ERROR	116.672	66.	1.768		
TOTAL	164.012	81.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	226.371	7.	32.339	3.821	0.002
TREATMENT MEANS	11.723	1.	11.723	1.385	0.243
HETEROGENEITY OF REGRESSION	19.287	7.	2.755	0.326	0.940
ERROR	558.619	66.	8.464		
TOTAL	816.000	81.			

ADJUSTED MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	MAJ.PRAC	FEW.PRACT	TOTAL
ACHIEVEMENT TEST2	9	21.64	19.33	20.54
ACHIEV REL CONCEPT	10	10.69	9.17	9.96
ACHIEV IRREL CONCE	11	4.18	4.03	4.11
ACHIEV COMPUTATION	12	9.37	8.59	9.00

TABLE 11

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	189.402	7.	27.057	4.387	0.002
TREATMENT MEANS	0.842	3.	0.281	0.045	0.987
HETEROGENEITY OF REGRESSION	212.999	21.	10.143	1.645	0.109
ERROR	172.691	28.	6.168		
TOTAL	575.934	59.			

DEPENDENT VARIABLE -- ACHIEV IRREL CONCE

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	45.945	7.	6.564	2.950	0.019
TREATMENT MEANS	14.237	3.	4.746	2.133	0.118
HETEROGENEITY OF REGRESSION	27.913	21.	1.329	0.597	0.887
ERROR	62.304	28.	2.225		
TOTAL	150.400	59.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	250.322	7.	35.760	5.283	0.001
TREATMENT MEANS	5.258	3.	1.753	0.259	0.854
HETEROGENEITY OF REGRESSION	160.218	21.	7.629	1.127	0.373
ERROR	189.538	28.	6.769		
TOTAL	605.336	39.			

ADJUSTED MEANS VARIABLES BY GROUPS

VARIABLE NAME	NO.	MANYMANY	MANYFEW	FEWMANY	FEWFEW	TOTAL
ACHIEV PEL CONCEPT	10	10.70	10.80	10.62	10.46	10.63
ACHIEV IPSEL CONCE	11	5.34	4.12	4.35	3.97	4.40
ACHIEV COMPUTATION	12	10.85	10.12	10.04	10.39	10.33

TABLE 12

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	189.402	7.	27.057	3.722	0.003
TREATMENT MEANS	0.579	1.	0.579	0.080	0.779
HETEROGENEITY OF REGRESSION	66.088	7.	9.441	1.299	0.273
ERROR	319.865	44.	7.270		
TOTAL	575.934	59.			

DEPENDENT VARIABLE -- ACHIEV IREL CONCS

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	45.945	7.	6.564	3.110	0.009
TREATMENT MEANS	4.659	1.	4.659	2.207	0.144
HETEROGENEITY OF REGRESSION	6.925	7.	0.989	0.469	0.852
ERROR	92.870	44.	2.111		
TOTAL	150.400	59.			

EPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	250.322	7.	35.760	5.089	0.000
TREATMENT MEANS	0.824	1.	0.824	0.117	0.734
HETEROGENEITY OF REGRESSION	45.033	7.	6.433	0.916	0.504
ERROR	309.157	44.	7.026		
TOTAL	605.336	59.			

ADJUSTED MEANS VARIABLES BY GROUPS

VARIABLE NAME	NO.	MANYEXAM	FEWEXAM	TOTAL
CHIEV REL CONCEPT	10	10.75	10.54	10.63
CHIEV IRREL CONCE	11	4.74	4.13	4.40
CHIEV COMPUTATION	12	10.47	10.22	10.33

TABLE 13

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	199.402	7.	27.057	4.325	0.001
TREATMENT MEANS	0.043	1.	0.043	0.007	0.935
HETEROGENEITY OF REGRESSION	111.205	7.	15.886	2.539	0.028
ERROR	275.284	44.	6.256		
TOTAL	575.934	59.			

DEPENDENT VARIABLE -- ACHIEV IRREL CONCE

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	45.945	7.	6.564	3.222	0.008
TREATMENT MEANS	7.900	1.	7.900	3.878	0.055
HETEROGENEITY OF REGRESSION	6.918	7.	0.988	0.485	0.840
ERROR	89.637	44.	2.037		
TOTAL	150.400	59.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	250.322	7.	35.760	5.761	0.000
TREATMENT MEANS	0.290	1.	0.290	0.047	0.830
HETEROGENEITY OF REGRESSION	81.584	7.	11.655	1.877	0.096
ERROR	273.140	44.	6.208		
TOTAL	605.336	59.			

ADJUSTED MEANS VARIABLES BY GROUPS

VARIABLE NAME	NO.	MANYPRAC	FEWPRACT	TOTAL
ACHIEV REL CONCEPT	10	10.66	10.61	10.63
ACHIEV LABEL CONCE	11	4.81	4.02	4.40
ACHIEV COMPUTATION	12	10.41	10.26	10.33

TABLE 14

CONCEPT

SCALE STATISTICS:

NUMBER OF CASES	=	75
NUMBER OF ITEMS	=	5
MEAN TOTAL SCORE	=	4.960
STANDARD DEVIATION	=	2.265
CRONBACH'S ALPHA	=	0.665
ERROR OF MEASUREMENT	=	1.312

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
1	0.547	0.554	0.482	1.333
12	0.627	0.635	0.343	1.333
17	0.640	0.658	0.560	2.667
13	0.427	0.438	0.430	2.667
14	0.453	0.466	0.165	2.667
15	0.400	0.411	0.423	2.667
16	0.440	0.458	0.304	4.000
3	0.707	0.716	0.584	1.333
10	0.720	0.730	0.694	1.333

COMPUTATION

SCALE STATISTICS:

NUMBER OF CASES	=	75
NUMBER OF ITEMS	=	17
MEAN TOTAL SCORE	=	7.787
STANDARD DEVIATION	=	3.210
CRONBACH'S ALPHA	=	0.741
ERROR OF MEASUREMENT	=	1.632

ITEM STATISTICS:

ITEM	P'S	ADJ. P'S	N.S. BIS	PERCENT NT
2	0.627	0.644	0.305	2.667
11	0.827	0.838	0.626	1.333
5	0.373	0.373	0.316	0.0
6	0.707	0.768	0.554	8.000
7	0.880	0.892	0.454	1.333
8	0.867	0.867	0.444	0.0
4	0.587	0.587	0.236	0.0
24	0.373	0.406	0.304	8.000
9	0.547	0.547	0.254	0.0
18	0.373	0.412	0.627	9.333
22	0.413	0.463	0.597	10.667
19	0.147	0.153	0.748	4.000
20	0.120	0.132	0.744	9.333
21	0.173	0.181	0.709	4.000
23	0.453	0.479	0.294	5.333
25	0.067	0.088	0.676	24.000
26	0.253	0.288	0.365	12.000

TABLE 15

RAW SCORE
MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
MISSING WORD 1	2	3.50	3.62	3.53	3.30	3.48
MISSING WORD 2	3	3.50	3.62	7.47	3.52	4.36
MISSING WORD 3	4	3.14	3.24	2.93	3.00	3.08
MISSING WORD 4	5	3.93	4.10	3.73	4.43	4.10
MISSING WORD 5	6	20.86	18.33	22.13	18.70	19.71
ARITH OPERATION	7	6.43	6.90	6.67	7.17	6.85
COMPUTATION - RATI	8	14.00	12.19	13.60	12.61	12.96
ACHIEVEMENT TEST2	9	12.43	14.95	14.27	11.30	13.18
ACHIEV REL CONCEPT	10	4.36	6.00	5.60	4.48	5.12
ACHIEV COMPUTATION	11	8.07	8.95	8.67	6.83	8.05

STANDARD DEVIATIONS
VARIABLES BY GROUPS

VARIABLE NAME	NO	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
MISSING WORD 1	2	0.65	0.74	0.52	0.70	0.67
MISSING WORD 2	3	0.85	0.59	15.65	0.95	7.12
MISSING WORD 3	4	0.66	1.14	0.88	1.09	0.98
MISSING WORD 4	5	1.59	1.58	1.71	1.27	1.51
MISSING WORD 5	6	4.85	7.57	6.91	6.34	6.64
ARITH OPERATION	7	2.56	2.43	1.76	2.33	2.27
COMPUTATION - RATI	8	3.46	5.12	2.80	4.18	4.10
ACHIEVEMENT TEST2	9	5.80	4.65	3.41	5.82	5.21
ACHIEV REL CONCEPT	10	2.68	1.97	1.76	2.47	2.32
ACHIEV COMPUTATION	11	3.56	3.32	2.44	3.75	3.41

CORRELATION MATRIX FOR TOTAL

		2	3	4	5	6	7	8	9	10	11
MISSING WORD 1	2	1.00	0.11	0.15	0.41	0.28	-0.02	0.20	0.21	0.13	0.2
MISSING WORD 2	3		1.00	0.12	-0.01	0.06	-0.04	-0.02	0.05	0.14	-0.0
MISSING WORD 3	4			1.00	0.11	0.10	0.15	0.17	0.22	0.36	0.0
MISSING WORD 4	5				1.00	0.30	0.34	0.14	0.20	0.22	0.1
MISSING WORD 5	6					1.00	0.23	0.38	0.49	0.40	0.4
ARITH OPERATION	7						1.00	0.42	0.51	0.44	0.4
COMPUTATION - RATI	8							1.00	0.46	0.38	0.4
ACHIEVEMENT TEST2	9								1.00	0.86	0.9
ACHIEV REL CONCEPT	10									1.00	0.5
ACHIEV COMPUTATION	11										1.00

SAMPLE SIZE = 73

TABLE 16

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	148.214	7.	21.173	7.205	0.000
TREATMENT MEANS	40.249	3.	13.416	4.565	0.008
HETEROGENEITY OF REGRESSION	76.944	21.	3.664	1.247	0.266
ERROR	120.484	41.	2.939		
TOTAL	385.891	72.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	363.854	7.	51.979	6.989	0.000
TREATMENT MEANS	46.101	3.	15.367	2.066	0.120
HETEROGENEITY OF REGRESSION	120.877	21.	5.756	0.774	0.732
ERROR	304.949	41.	7.438		
TOTAL	835.781	72.			

ADJUSTED
MEANS
VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	GROUP 3	GROUP 4	TOTAL
ACHIEV REL CONCEPT	10	4.27	6.18	5.39	4.51	5.12
ACHIEV COMPUTATION	11	7.91	9.19	8.13	7.06	8.05

TABLE 17

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	148.214	7.	21.173	5.846	0.000
TREATMENT MEANS	4.805	1.	4.805	1.327	0.254
HETEROGENEITY OF REGRESSION	26.439	7.	3.777	1.043	0.412
ERROR	206.433	57.	3.622		
TOTAL	385.891	72.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	363.854	7.	51.979	7.057	0.000
TREATMENT MEANS	23.512	1.	23.512	3.192	0.079
HETEROGENEITY OF REGRESSION	28.581	7.	4.083	0.554	0.789
ERROR	419.835	57.	7.366		
TOTAL	835.781	72.			

ADJUSTED MEANS VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	TOTAL
ACHIEV REL CONCEPT	10	5.40	4.87	5.12
ACHIEV COMPUTATION	11	8.67	7.49	8.05

TABLE 18

DEPENDENT VARIABLE -- ACHIEV REL CONCEPT

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	148.214	7.	21.173	5.313	0.000
TREATMENT MEANS	3.736	1.	3.736	0.938	0.337
HETEROGENEITY OF REGRESSION	6.788	7.	0.970	0.243	0.972
ERROR	227.153	57.	3.985		
TOTAL	385.891	72.			

DEPENDENT VARIABLE -- ACHIEV COMPUTATION

ANALYSIS OF COVARIANCE

SOURCE OF VARIATION	ADJ. SS	DF	ADJ. MS	F	P
REGRESSION	363.854	7.	51.979	7.539	0.000
TREATMENT MEANS	0.080	1.	0.080	0.012	0.915
HETEROGENEITY OF REGRESSION	78.831	7.	11.262	1.633	0.145
ERROR	393.016	57.	6.895		
TOTAL	835.781	72.			

ADJUSTED MEANS VARIABLES BY GROUPS

VARIABLE NAME	NO.	GROUP 1	GROUP 2	TOTAL
ACHIEV REL CONCEPT	10	4.82	5.32	5.12
ACHIEV COMPUTATION	11	8.01	8.08	8.05

APPENDIX 2

Number of Problems

For Each Variation

Concepts	Examples		Practice	
	Few	Many	Few	Many
Section 1				
Experiment	15	18	1	2
Predictable & Chance	3	4	8	12
Event	5	5	3	9
Equally Likely	4	6	1	4
Probability	12	16	7	21
Section 2				
Complex Experiment	4	5	6	8
Trial	4	6	2	2
Tree Diagram	2	2	1	3
Table of Outcomes	3	3		
Probability	2	5	13	23

Appendix 3

Topics, Concepts, Dimensions, and Skills. Corresponding test items are also designated.

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Revision of the list of Concepts along with Relevant and Irrelevant dimensions.

SECTION 1

Experiment A well defined act or process that leads to a well defined set of possible outcomes.

Relevant Dimensions:

1, 7	1. Act and Set of possible outcomes.
2	2. Different sets of possible outcomes imply different experiment.
3, 16	3. There exists only one outcome per act.

Irrelevant Dimensions:

4, 5	1. Number of coutcomes possible.
6, 11, 12	2. Chance that an outcome will happen.
8	3. Order of the outcomes.

Skills:

9, 10	1. List the set of outcomes.
12, 13	2. Count the number of possible outcomes.

Predictable If the result of an act is certain, then it is predictable.

Relevant Dimension

14, 24	1. Certainty that the result will or will not happen.

Chance The result of an act is uncertain.

15, 23

Event A subset of the set of all possible outcomes of an experiment.

Relevant Dimesion:

19, 20	1. Subset
26, 27	2. Set of all possible outcomes

Irrelevant Dimension

19, 20	1. The number of elements in the subset
Skill: 17, 18	List

Equally Likely An undefined description of the outcomes of an experiment.

Relevant Dimesions

1. More likely
2. Less likely

Irrelevant

28	1. Number of outcomes

Skill:

22, 28	1. Be able to distinguish outcomes which are equally likely.
29, 30	

Probability The probability of an event if the fraction:

$$\frac{\text{number of outcomes in the event}}{\text{number of possible outcomes}}$$

Relevant Dimensions

31, 32 1. $0 \leq P(\text{event}) \leq 1$
33, 34 2. If P(event) is equal to 0 or 1, then it is predictable
21, 25 3. Chances that an event will happen

Skills

35, 36, 37 1. Use of P(event) notation
38, 39 2. Compute (event) for given experiments.

SECTION 2

Complex Experiment An experiment made up of two or more simple experiments.

Relevant Dimensions

1 1. At least two simple experiments.
2. Outcomes are ordered n-tuples where n = the number of simple experiments.

Irrelevant Dimensions

10 1. Number of simple experiments as long as $n \geq 2$.
3 2. What the simple experiments are.

Skill

2, 6, 7, 11 1. Use of appropriate notation for outcomes.
5 2. Count the outcomes.

Trial A trial is a repetition of an experiment.

Relevant Dimension

17 1. Repetion of the same experiment
12 2. Each trial has one outcome

Irrelevant Dimension

1. Outcomes of different trials
2. Number of trials in the past

Skill

8 1. Count number of trials

Tree Diagram

Relevant Dimension

1. Simple experiments
4 2. All possible outcomes per simple experiment

Irrelevant Dimensions

1. Same as those for a complex experiment

Skill

4, 9, 18, 22, 24 1. Draw a tree diagram

Table of Outcomes

Relevant Dimensions

13 1. Each row represents the outcome of one trial
14 2. Each column represents the result of a simple experiment
15 3. Number of rows
16 4. Number of columns

Skills

1. Understand notation

Probability for Complex Experiments

Comments for probability apply.

Skills:

Compute the probability for complex experiments involving descriptions such as

19, 25 1. "at least
20, 26 2. "exactly"
21 3. "more than"
23 4. "not"

*Numbers indicate the test items designed to examine the specific dimension or skills.

Test Scales Designated by Item

Test 1

Scale	Items
Relevant Concept	1, 2, 3, 7, 14, 15, 16, 19, 20, 21, 23, 24, 25, 26, 27, 31, 32, 33, 34
Irrelevant Concept	4, 5, 6, 8, 11, 12, 19, 20, 24, 28
Computation	9, 10, 12, 13, 17, 18, 22, 28, 29, 30, 35, 36, 37, 38, 39

Test 2

Relevant Concept	1, 12, 13, 14, 15, 16, 17
Computation	2, 4, 5, 6, 7, 8, 9, 11, 18, 19, 20, 21, 22, 23, 24, 25, 26

APPENDIX 4

Achievement Tests 1 and 2

SECTION I

ACHIEVEMENT TEST

INSTRUCTIONS

This is a test about probability. You will have 45 minutes to finish the test. Read the questions carefully and answer them to the best of your ability.

Please try all the questions. However, if you find some questions too hard, leave them until you have answered the easier ones. Then you may go back and try the harder ones. Try to do the best you can.

Here are some sample questions:

S1. In the experiment of tossing a fair coin, P(Heads) =

(A) 1/2 (B) 3/4 (C) 1 (D) 1/4 (E) None of these

S2. What is the set of possible outcomes in the experiment of tossing a die?

_____

All answers should be placed on the answer sheet provided. For a question like S1 place an X through the letter on the answer sheet that corresponds to your answer to the question. For a question like S2 write in your answer to the question in the space provided on the answer sheet.

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ACHIEVEMENT TEST

SECTION 1

1. Which of the following is (are) experiment(s)?

   (i) Toss a nickel and record the face that lands upward

   (ii) Drop a piece of chalk on the floor and count the number of pieces it breaks into

   (iii) Pick up the phone and record the time lapse before you get the dial tone.

   (A) i only

   (B) both i and ii

   (C) both i and iii

   (D) all three are experiments

   (E) None of the three is an experiment

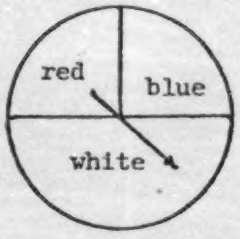
2. Suppose you have a deck of five cards: the nine and ten of spades, and the two, three, and five of hearts. A boy draws a card from the deck, records the number, and returns the card. A girl draws a card from the deck, records the suit and returns the card. Are the boy and girl performing the same experiment?

   (A) Yes

   (B) No

   (C) Neither are doing an experiment

3. Using the spinner at the right, which of the following results could <u>not</u> occur in 100 spins?



   (A) 25 red, 25 blue, and 50 white

   (B) 50 red and 50 white

   (C) 25 red and 75 blue

   (D) 100 white

   (E) 30 red, 30 blue, and 60 white

4. In any experiment, the maximum number of outcomes in the set of possible outcomes is

(A) 0

(B) 1

(C) 6

(D) 1000

(E) As many as you want

5. Which of the following is (are) experiment(s)?

Drawing and recording one card from a deck of

(i) 52 cards

(ii) 15 cards

(iii) 7 cards

(vi) 1 card

(A) i only

(B) ii only

(C) i and iii

(D) i, ii, and iii

(E) i, ii, iii and iv

6. In experiment A, you have a deck which has 13 spades and 1 heart. You draw a card from the deck and record its suit.

In experiment B, you have a deck which has 13 spades and 13 hearts. You draw a card from the deck and record its suit.

Which of the following is correct?

(A) the sets of possible outcomes for experiment A and B are the same

(B) the sets of possible outcomes for experiment A and B are different

(C) neither of the above

7. In each example below, both the act and set of possible outcomes is given. Which of the experiments are the same?

(i) flipping a coin and recording Heads or Tails

(ii) throwing a die and recording whether it lands on the Floor or Table

(iii) flipping a coin and recording whether it lands on the Floor or Table

(A) i and ii

(B) i and iii

(C) ii and iii

(D) they are all the same experiment

(E) they are all different experiments

8. Three experiments have the same act and the sets of possible outcomes for the three experiments are:

(i) {1,2,3,4,5,11}

(ii) {2,3,1,4,5,11}

(iii) {3,11,5,2,1, 4}

Which of the experiments are equal?

(A) i and ii

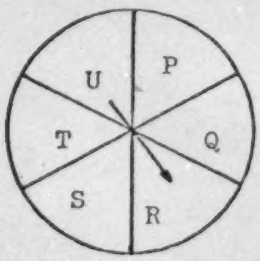
(B) i and iii

(C) ii and iii

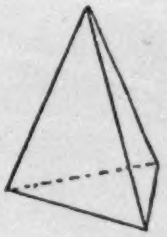
(D) i, ii, and iii

(E) none of the above

9. List the possible outcomes of spinning the spinner below.



10. The letters A, B, C, D are printed on the four faces of pyramid shaped die. It is tossed in the air and allowed to fall freely. Let the outcome be the letter on the bottom face. Write the set of possible outcomes.



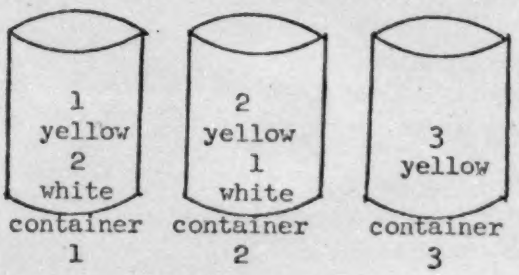
_____

11. Each of the three cans on the right contains three tennis balls.

In experiment 1, you draw a ball from container 1.

In experiment 2, you draw a ball from container 2.

In experiment 3, you draw a ball from container 3.



__ 'Yellow ball' is an outcome for which of the experiments?

(A) Experiment 1

(B) Experiment 2

(C) Experiment 3

(D) Experiment 1 and 2

(E) Experiment 1, 2 and 3

12. How many outcomes are possible for the experiment of throwing and recording the color of a cube that has three of its faces colored blue, and three colored green?

(A) 1

(B) 2

(C) 4

(D) 6

(E) None of the above

13. A group of three boys and five girls want to decide who will be the group leader by drawing a name from a box. If the name of each person is written on a piece of paper and put in the box, how many outcomes are in the experiment of drawing a name from the box?

(A) 2

(B) 3

(C) 5

(D) 8

(E) None of the above

14. If an event of an experiment is predictable, then the P(event) is

(A) 0

(B) 1

(C) between 0 and 1

(D) 0 or 1

(E) none of the above

15. Three hats are in a dark closet. Two belong to Mr. Smith and the other to his friend. Mr. Smith reaches in the closet and draws two hats. Which of the following statements correctly describes the chances that Mr. Smith will draw his own hats.

(A) He is certain to draw both of his own hats.

(B) He is certain not to draw both of his own hats.

(C) He may draw both of his own hats.

(D) None of the above

16. A pyramid shaped die has four faces with the numbers 1, 2, 3, and 4 marked on them. If you toss the die and record the number that lands downward, the set of possible outcomes is $\{1,2,3,4\}$. How many outcome(s) will you get each time you toss the die?

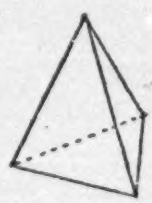
(A) one

(B) two

(C) three

(D) four

(E) the number will vary



17. A box has nine plastic blocks numbered 1 - 9, all the same shape and size. List the outcomes of the event of drawing a block numbered "less than 7."

_____

18. A spinner has 8 sections numbered 5 to 12. List the outcomes of the event "greater than 8."

_____

19. An experiment has the set of outcomes {1,3,5,7,9} . Which one of the following is <u>not</u> an event of the experiment?

(A) {1,3,5,7,9}

(B) {7,3}

(C) {1,3,7,9}

(D) ∅

(E) None of the above

20. An experiment has the set of outcomes {red, white, brown} . Which of the following is an event of the experiment?

(A) {black}

(B) {red, white, brown, black}

(C) {1,2,3}

(D) {red, white, brown}

(E) None of the above

21. The probability of drawing a king from a deck of cards is 1/13. Which of the following <u>must</u> be true?

(A) there are 52 cards

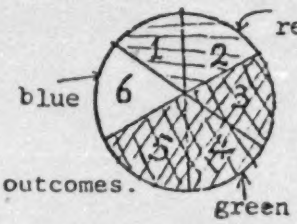
(B) you are certain to get a king

(C) there is one chance out of 13 that you will draw a king

(D) there are 13 chances out of 13 that you will draw a king

(E) none of the above

22. In experiment A, you spin the spinner and find out which one of the six numbers, the pointer stops at. In experiment B, you spin the spinner and find what color the pointer stops at. Which of the following statements is correct?



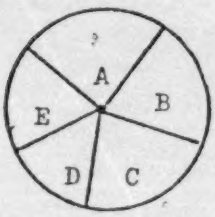
(A) Experiment A has equally likely outcomes.

(B) Experiment B has equally likely outcomes.

(C) Both experiment A and B have equally likely outcomes.

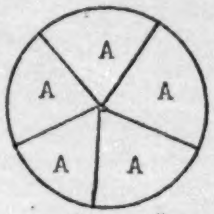
(D) Neither A nor B has equally likely outcomes.

The next two questions refer to the following four spinners.

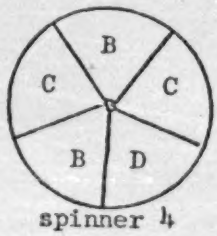


spinner 1

spinner 2



spinner 3



spinner 4

23. If you spin spinner 4, the event "B"

(A) may happen

(B) is certain to happen

(C) will not happen

(D) None of the above

24. For which spinner(s) is the event "A" predictable?

(A) Spinners 1 and 2

(B) Spinners 3 and 4

(C) Spinner 3

(D) Spinner 4

(E) None of the above

25. In an experiment, the probability of event A is 2/10, the probability of event B is 3/10, and the probability of event C is 1/10. Which of the following is true?

(A) There are more chances to get event A

(B) There are more chances to get event B

(C) There are more chances to get event C

(D) A and B are equally likely events

(E) None of the above

26. For which of the following sets is the set {2,4,6} an event?

(A) {1,2,3,4,5}

(B) {1,3,5,7,9}

(C) {2,4,6,8,10}

(D) $\emptyset$

(E) None of the above

27. An experiment has 3 possible outcomes. The number of elements contained in any event of the experiment must be less than

(A) 0

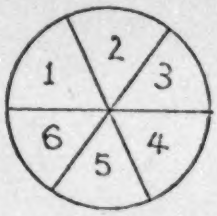
(B) 2

(C) 3

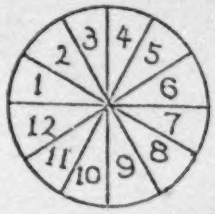
(D) 4

(E) None of the above

28. Each of the spinners below represents an experiment. Which of the experiments has (have) equally likely outcomes?



spinner 1



spinner 2

spinner 3

(A) Spinner 1 has equally likely outcomes

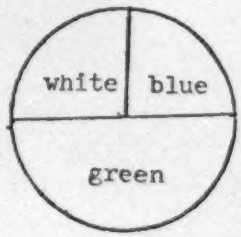
(B) Both spinners 1 and 2 have equally likely outcomes

(C) Spinners 1 and 3 have equally likely outcomes

(D) Spinners 1,2, and 3 have equally likely outcomes

(E) None of the above

29. If you play a game with this spinner to the right. you are most likely to get



(A) More blues than greens

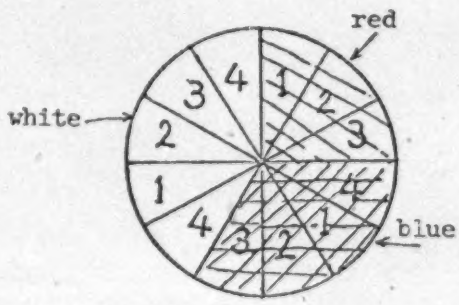
(B) More whites than blues

(C) More whites than greens

(D) The same number of blues and whites

(E) The same number of greens and whites

30. In the spinner on the right, each of the 12 small sectors has the same area. Which of the following sets of outcomes is equally likely?



(A) {1,2,3,4}

(B) {blue, red, white}

(C) {red 1, blue 4, white 4}

(D) None of the above

31. A bag contains several marbles. Two of the marbles are orange. Which of the following must be true?

(A) P(Orange) = 0

(B) P(Orange) is smaller than 0

(C) P(Orange) is greater than 0

(D) P(Orange) = -2/5

(E) None of the above

32. A spinner is divided into eight equal sections using three different colors, Blue, Orange, and Green. Which of the following is true?

(A) P(Orange) = 1

(B) P(Orange) is greater than 1

(C) P(Orange) is smaller than 1

(D) P(Orange) = 10/8

(E) None of the above

33. For a certain bag of marbles, the event "orange" is sure to happen. Which of the following must be true?

(A) P(Orange) = 0

(B) P(Orange) = 1

(C) P(Orange) is greater than 0

(D) P(Orange) is smaller than 1

(E) None of the above

34. From a bag of marbles, a blue marble will never be drawn. Which of the following must be true?

(A) P(Blue) = 0

(B) P(Blue) = 1

(C) P(Blue) is greater than 0

(D) P(Blue) is smaller than 1

(E) None of the above

35. Using the P( ) notation, instead of writing "the probability of less than 5", you would write

_____

36. A bag contains 12 tennis balls. Some are red and some are white. P(White) = 2/3. How many red balls are in the bag?

(A) 1

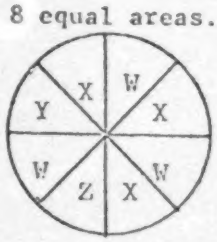
(B) 8

(C) 4

(D) 10

(E) None of the above

37. The face of the spinner below is dividied into 8 equal areas. What is P(X)?



_____

38. There are 9 possible equally likely outcomes for an experiment. The event "Blue" contains 6 of the possible outcomes. What is P(Blue)?

_____

39. A bag contains marbles of three different colors: Blue, Green, and Red. The probability of drawing a blue marble is 2/7, and the probability of drawing a green marble is 3/7. What is the probability of drawing a red marble?

(A) 1/7

(B) 2/7

(C) 3/7

(D) 4/7

(E) None of the above

SECTION II

ACHIEVEMENT TEST

INSTRUCTIONS

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S1. In the experiment of tossing a fair coin, P(Heads) =

(A) 1/2 (B) 3/4 (C) 1 (D) 1/4 (E) None of these

S2. What is the set of possible outcomes in the experiment of tossing a die?

_____

All answers should be placed on the answer sheet provided. For a question like S1 place an X through the letter on the answer sheet that corresponds to your answer to the question. For a question like S2 write in your answer to the question in the space provided on the answer sheet.

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SECTION II

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S2. What is the set of possible outcomes in the experiment of tossing a die?

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You may turn the page and begin.

ACHIEVEMENT TEST

SECTION II

1. A complex experiment consists of at least _____ simple experiments.

   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) none of the above

2. A complex experiment consists of 4 simple experiments. Which of the following could be an outcome of the experiment?

   (A) 1
   (B) (1,3)
   (C) (1,1,3,1)
   (D) (1,1,3)
   (E) none of the above

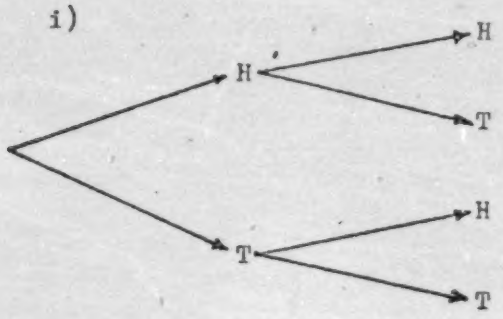
3. Which of the following is (are) complex experiment(s)?

   (i) Flipping a die and a coin
   (ii) Spinning a spinner and flipping a coin
   (iii) Picking a marble from a bag and tossing a die

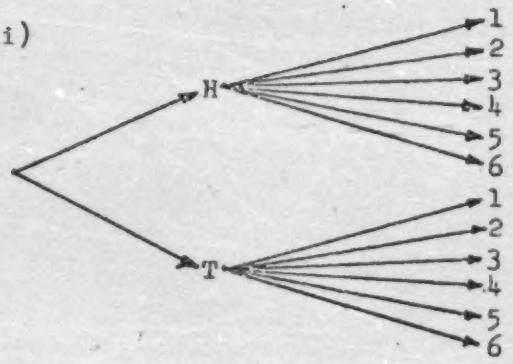
   (A) only i
   (B) i and ii
   (C) i and iii
   (D) ii and iii
   (E) all three are complex experiments

4. Which of the following is correct?

i)

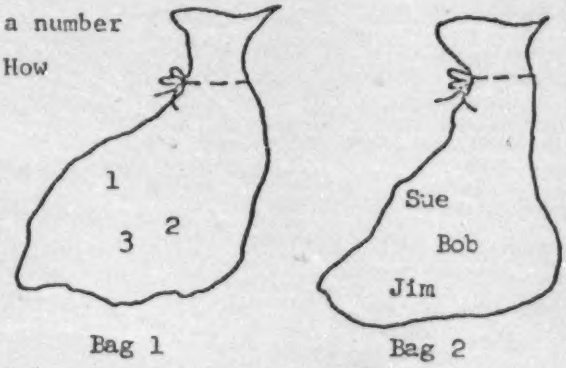


ii)



(A) only i is a tree diagram
(B) only ii is a tree diagram
(C) both i and ii are tree diagrams
(D) none of them is tree diagram
(E) don't know

5. Consider the experiment of drawing a number from bag 1 and a name from bag 2. How many possible outcomes are in the experiment?



(A) 3
(B) 6
(C) 9
(D) 12
(E) none of the above

6. For the same experiment, list the outcome(s) in the event "even number and a girl."

7. In the complex experiment of tossing a die, flipping a coin, and picking a number from a bag containing numbers 1 to 10, which of the following is the outcome of getting a 'one' on the die, a 'head' on the coin, and a '3' from the bag?

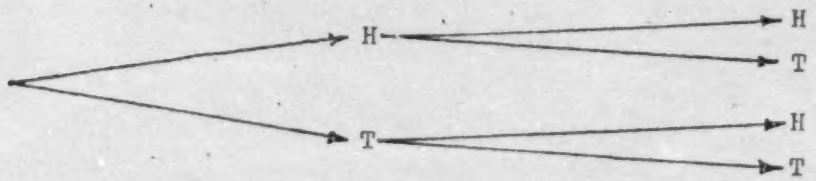
(A) (1, H, 1)
(B) (3, H, 1)
(C) (1, T, 3)
(D) (1, H, 3)
(E) none of the above

8. We have just performed the experiment of tossing a coin and rolling a die. How many trials of the experiment are shown in the table below?

coin	die
H	5
H	3
T	1
T	6
H	3

(A) 2
(B) 5
(C) 6
(D) 12
(E) none of the above

9. The following is a tree diagram of tossing a penny and a nickel:



How many possible outcomes will there be if a dime is tossed in addition to the penny and the nickel?

(A) 2
(B) 4
(C) 6
(D) 8
(E) none of the above

10. What is the maximum number of simple experiments in a complex experiment?

(A) 2
(B) 3
(C) 4
(D) 10
(E) as many as you want

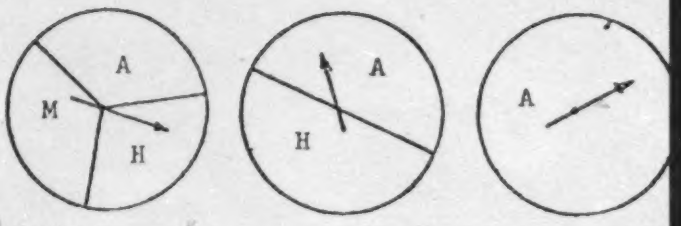
11. If (H,H,T,H) is an outcome of a complex experiment then the experiment consists of how many simple experiment(s)?

(A) 1
(B) 2
(C) 3
(D) 4
(E) none of the above

12. How many outcome(s) will there be in 30 trials of an experiment?

(A) 1
(B) 30
(C) 60
(D) 100
(E) as many as you want

The following is the table of possible outcomes for the experiment of spinning the three spinners on the right.



spinner 1 spinner 2 spinner 3

spinner 1	spinner 2	spinner 3
M	A	A
M	H	A
A	A	A
A	H	A
H	A	A
H	H	A

Question 13 to 16 refer to the above table.

13. Each row of the table represents

(A) one possible outcome of the complex experiment
(B) a set of outcomes for a simple experiment
(C) the set of possible outcomes of the complex experiment
(D) none of the above

14. Each column of the table represents

(A) one possible outcome of the complex experiment
(B) a set of outcomes for a simple experiment
(C) the set of possible outcomes of the complex experiment
(D) none of the above

15. The number of possible outcomes for the experiment is equal to

(A) the number of rows
(B) the number of columns
(C) the number of rows plus the number of columns
(D) none of the above

16. The number of simple experiments in a complex experiment is equal to

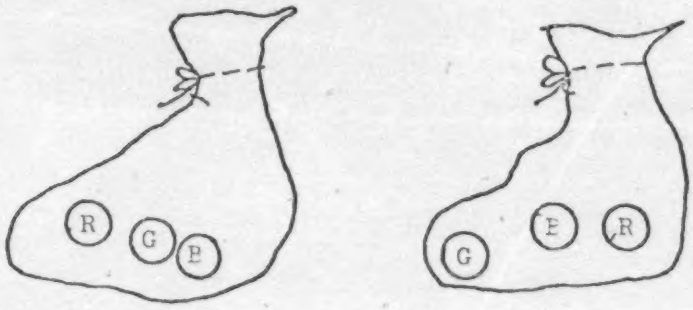
(A) the number of rows
(B) the number of columns
(C) the number of rows plus the number of columns
(D) none of the above

17. The number of _____ of the experiment refers to the number of times we repeat the experiment.

(A) trials
(B) events
(C) outcomes
(D) chances
(E) none of the above

18. Draw a tree diagram (on the back of your answer sheet) of the outcomes of tossing a die and a penny.

In an experiment, you draw a marble from each of the two bags below.



Questions 19 to 21 refer to the experiment above.

19. P (at least one red marble) =

(A) 1/6
(B) 2/6
(C) 1/9
(D) 4/9
(E) 5/9

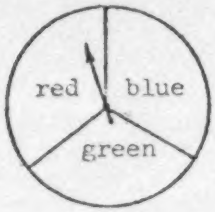
20. P (exactly one blue marble) =

(A) 1/6
(B) 2/6
(C) 1/9
(D) 4/9
(E) 5/9

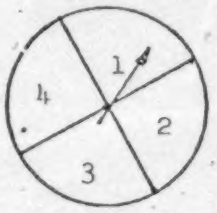
21. P (more than one green marble) =

(A) 1/6
(B) 2/6
(C) 1/9
(D) 4/9
(E) 5/9

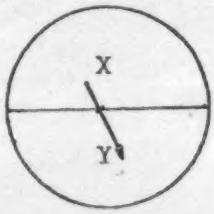
22. In a complex experiment you spin spinner 1, then spinner 2 and then spinner 3. Draw a tree diagram (on the back of your answer sheet) of the outcomes.



spinner 1



spinner 2

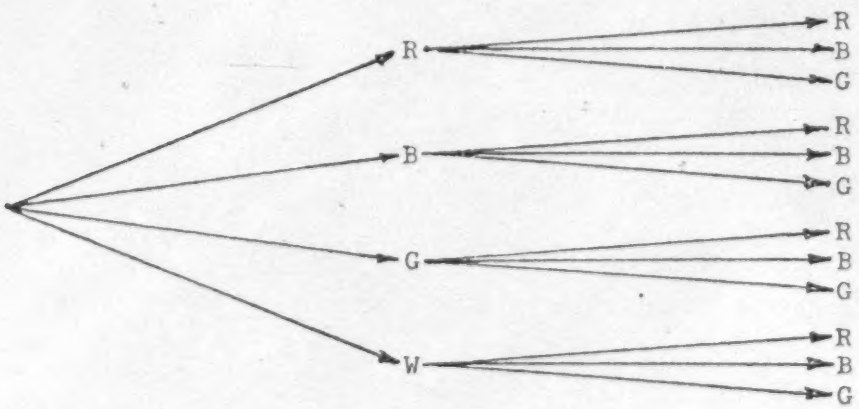


spinner 3

23. In an experiment the probability of the event "less than 5" is 1/9. What is P(<u>not</u> less than 5)?

(A) 0
(B) 1/9
(C) 5/9
(D) 8/9
(E) 1

24. Two identical pyramid shaped dice have 'R', 'B', 'G' and 'W' marked on their 4 faces. In a complex experiment you toss the two dice and record the face that lands downward. Is the below a tree diagram of the possible outcomes?



(A) yes
(B) no
(C) not enough information

25. In an experiment of tossing a penny, a nickel and a dime, what is the probability of getting at least one head?

(A) 1/6
(B) 3/6
(C) 3/8
(D) 6/8
(E) 7/8

26. In the same experiment, what is the probability of getting exactly 2 tails?

(A) 1/6
(B) 3/6
(C) 3/8
(D) 6/8
(E) 7/8